Ribbon Patterns with Geometry Software

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Abstract

Geometric patterns are an excellent learning resource for mathematics teachers for various levels of education. In this paper we explore a narrow group of patterns from Islamic art, so called ribbon patterns, and we will show how technology, in our case Geometer's Sketchpad, can be used to create these patterns in a very simple and attractive way.

Introduction

While developing any type of geometric patterns we learn and practice various elements of elementary geometry – with Gothic patterns we deal with geometry of circles, in geometric patterns from Islamic art we use extensively segments and polygons, in ancient Greek patterns we use frequently square and isometric grids, Thai patterns use frequently curves and spirals. All this brings us to the elementary geometry with compasses and ruler constructions. These constructions although often very simple are frequently repeated hundreds of times again and again. This tedious process can be usually simplified. For a project, in the classroom where we draw our patterns by hand, on a paper, we produce a fragment of the pattern and then we copy it to any place where we need it using tracing paper. This way we make our construction only once. The same result can be achieved using ceramic tiles. We have to create a specific tile, and make multiple copies of it. This way, ancient and contemporary artists produced a number of interesting works. We can find such works in all Muslim countries where an old architecture was preserved. In Turkey, and in particular in old capitals of this country, numerous mosques and madrasah are decorated with such tiles. The enclosed photo shows one of such patterns from the famous Green Mosque in Bursa. Here we see a mosaic made using copies of two tiles – left and right sides of the main motif. The whole pattern is just a geometric freeze created by translations on the main motif.
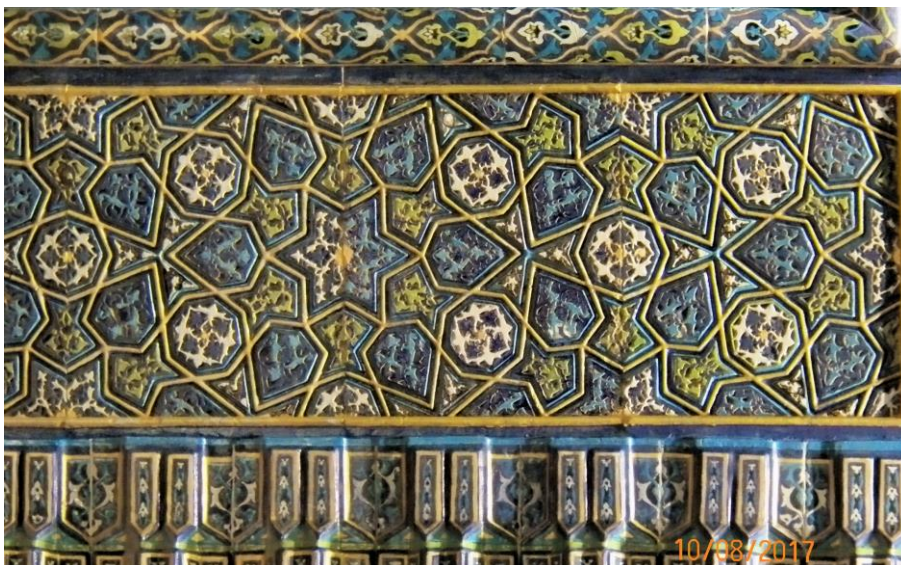

Fig. 1.1 Geometric decoration in Green Mosque in Bursa. The complex pattern was created using multiple copies of two tiles only.

Note – the tiles here are not square, they are derived from a rectangle with proportion of sides length equal to 1.73

Photo by Mirek Majewski

Custom tools in geometry software

There are at least two computer programs for geometry allowing us to mimic precisely the decorated tile technique. One of them is *Geometer's Sketchpad* and another one is *GeoGebra*. In both programs we can create complex geometric constructions, save them as custom tools, and then use these tools any time when we need them. Although both programs allow us to make such tools, there are some differences worth mentioning here. In Geometer's Sketchpad in one file we can develop any number of tools and after saving the file we make these tools available for all our later works. In GeoGebra tools created in one file can be saved in a file with specific extension *.ggt and loaded manually when we need them. In GSP a custom tool has a name only. In GeoGebra we can assign an icon to a tool and have its visual representation on GeoGebra tools menu. Before, we will proceed with the main topic of this paper let us to see how a custom tool can be created. We will use GSP to demonstrate this concept, but one can do precisely the same with GeoGebra.

Project 1. A square and hexagon project
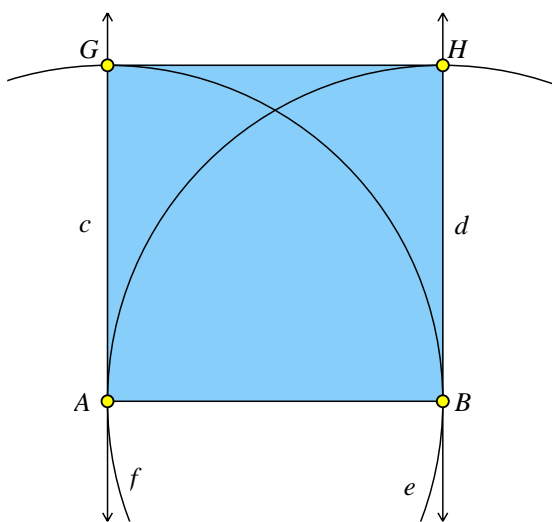
Ryc. 1. *Custom tool design for drawing a square (GSP)*

STEP 1: Draw a segment AB and two perpendicular lines to it 'c' and 'd'. Two circles 'e' and 'f' while intersecting with the perpendicular lines create points 'G' and 'H'. Points 'A', 'B', 'H' and 'G' mark corners of the square. The square can be marked by a polygon or by adding extra segments joining vertices of the square.

STEP 2: Select, while pressing Shift key, first the two points 'A', 'B', and then the remaining elements of the square, e.g. points 'G', 'H', the polygon, etc. Whatever you select now, later it will be part of the tool.

STEP 3: From the Custom Tool menu (left bottom key with arrow on it) choose 'Create new tool'. Give a name for this tool, e.g. 'my square', and this is all. The tool is ready to use.
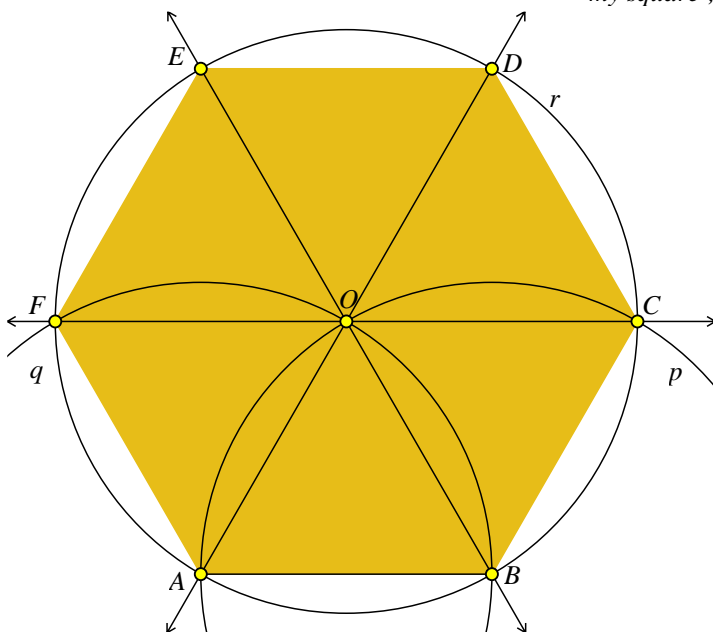
Fig. 2. *Custom tool to create hexagon (GSP)*

In exactly the same way like before, we start by drawing a segment AB, and then the three circles 'p', 'q' and 'r'.

Lines passing through the obtained points of intersection of circles mark the corner points of a hexagon.

By selecting points only and shown here polygon we make preparation to create the hexagon tool.

IMPORTANT NOTE

Any regular polygon can be created using various starting points. In our case these were points 'A' and 'B'. One can also construct a square or a hexagon from its diagonal, or a center and one of the corners.
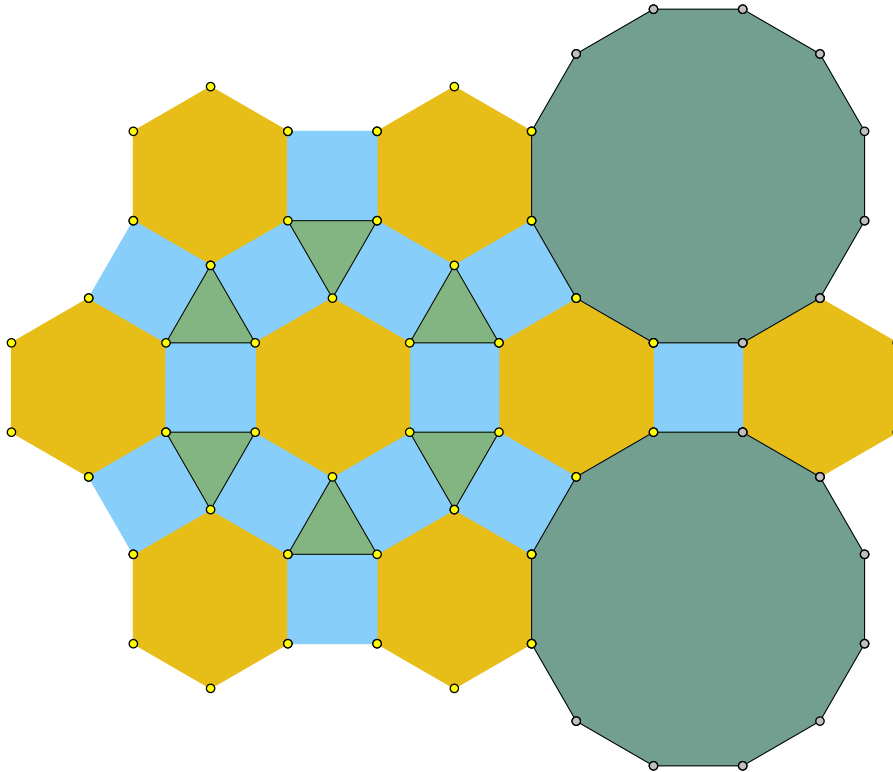
Fig 4. *A mosaic created with the use of four tools – a square, an equilateral triangle, a regular hexagon and regular dodecagon.*

The figure 3 shows how selection of elements in tool design influences what the tool produces. For square and hexagon tools we selected points and polygons only. We did not select sides of these polygons. While creating tools for the triangle and dodecagon I selected also their sides. As a result we see them in our mosaic.

Tools in geometry programs can be used for producing another tools, and these in another tools. For example now, we could add patterns to our tools and produce patterned tiles. Step-by-step using tools, very complex results can be achieved.

Ribbon patterns

Interlace patterns were very popular in the majority of cultures of the East and West. We rarely see them in Chinese lattice design (see [6]), often in Celtic art, and in many other cultures. In Islamic art interlace patterns have attained a special attention of artists both from the past as well as contemporary designers (see [4, 7, 10]). We can find them mentioned or analyzed in numerous books and research papers.

In this paper we will concentrate on a narrow class of interlace patterns that sometimes are referred to as ribbon patterns or braided borders. The word 'borders' describes perfectly the use of such patterns – borders to a page, borders to an entrance to a mosque, or to a larger pattern, or to an Islamic calligraphy. In eastern Islam these patterns are usually executed in wood or stone, they are simple, not painted or colored in any way. In western Islam these patterns obtained a life on their own. Usually they are very impressive, colorful, complex, and in many cases they are the main 'actors' of larger designs. We rarely find them in Central Asia. In the book by the famous Russian scientists – L.I. Rempel (see [8]) we see only a very few examples of such patterns.

In Morocco and Spain these patterns occupy a significant place. We see a number of them in Owen Jones book (see [7]) and in many other books devoted to arts of this region. These patterns can be admired in a few places in Alhambra, e.g. in the palace of Nazrides (see fig. 8). Some of them are rendered in clay and glazed using clear strong colors. Some other are rendered in raw clay only and carved additionally with intricate arabesque motives.

Sometimes the role if such patterns is very puzzling. For example in the Sheikh Zayed Mosque in Abu Dhabi, which is full of gorgeous decorations with semiprecious stones and complex curvilinear pentagonal or decagonal patterns, we suddenly see in one corner a very humble ribbon pattern rendered in wood. In this case we can ask what the reason was to use this pattern in such a place. It certainly looks completely out of place. Does it have a special meaning?

Fig. 5. *Ribbon pattern from Sheikh Zayed Mosque in Abu Dhabi*

This pattern was implemented in humble piece of wood. A number of similar patterns we can find in countless places of the Islamic world. Surprisingly, similar patterns but with round corners were frequently used in Celtic art.
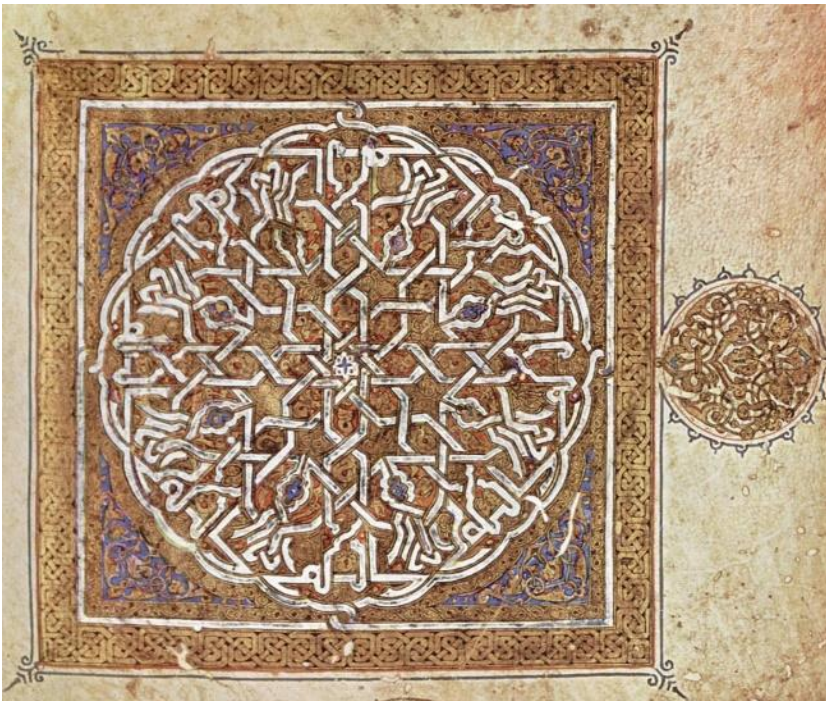
Photo by Mirek Majewski

Fig. 6. *An old ribbon pattern used as border for an old Qur'an carpet page, year 1182*

(fot. Wikimedia Commons)

Fig. 7. *Curvilinear ribbon pattern on the wall of one of the new mosques in Abu Dhabi.*

Photo by Mirek Majewski

Fig. 8. *Fragment of a ribbon pattern from Alhambra. Clay mosaic glazed using traditional colors from the western Islamic designs.*

Photo by Mirek Majewski

In this paper we will concentrate on ribbon patterns without additional elements and decorations. These will be mostly patterns similar to the motives shown in the figures 5, 6, and 8.

In a few books we can find attempts to analyze these patterns. However, none of them goes deep enough to give us a simple method to reconstruct such patterns or a way to create our own designs. A very good analysis of grids behind the ribbon patterns we can find in the book by David Wade (see [10]). We will use his analysis to develop a very simple method to create such patterns.
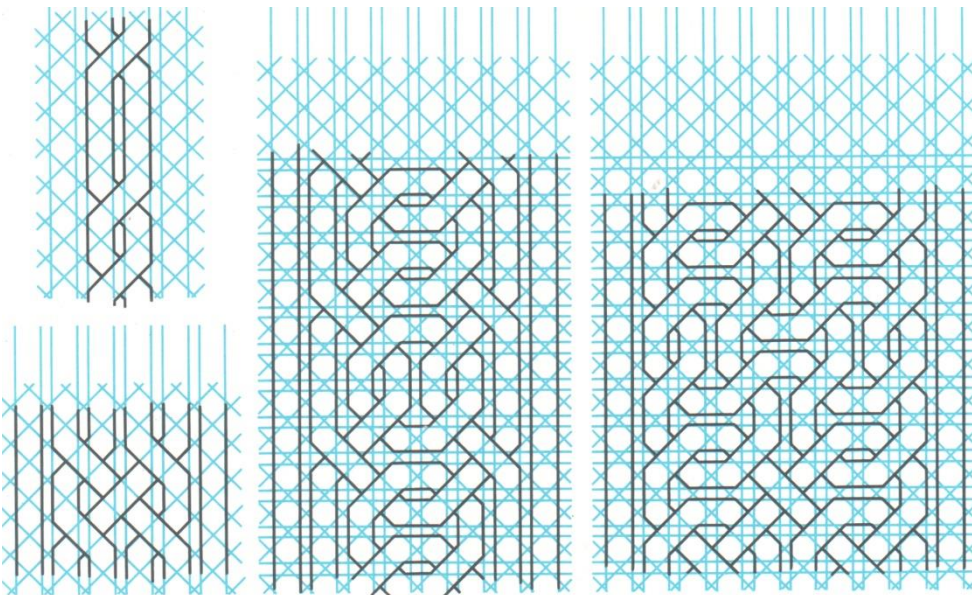

Fig. 9. *David's Wade analysis of a ribbon pattern*

Image courtesy of the Author

First of all we have to notice that these patterns usually have some borders, e.g. left and right border. Sometimes the borders are from all four sides. The second important observation is that most of the patterns are built out of slant paths, i.e. they go left-up or -down and right-up or -down. The horizontal

and vertical paths occur only along borders and in some other places which we discuss later. We can easily see that the slant paths go up or down in a regular way. While intersecting with other paths a given path goes alternately on top or below of the other path. The horizontal or vertical paths, with some rare exceptions, do not intersect. Finally we see that no more than two slant paths intersect in any point.

There is one thing that is not visible in Wade's analysis. If we draw points in empty slots between paths we will get a regular network of points (red points in fig. 10). There is more – if we draw points on each intersection of paths then we will get another network of points (yellow points in the next picture). Both networks of points – red and yellow – combined together form a regular square grid.
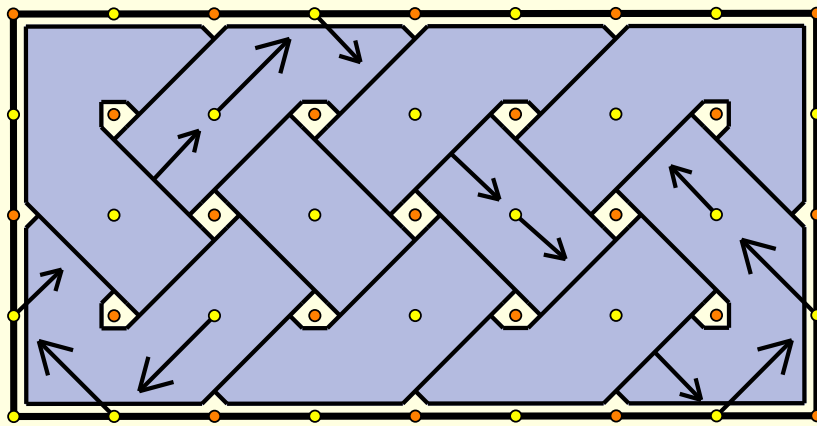
Fig. 10. *Flow of the paths through a simple ribbon pattern*

We start from any yellow point and go down left, or down right following arrows.

All paths of such a pattern go along the yellow points; they turn when they touch the border, and then again go along a slant line. This is all. Now we can develop our first ribbon pattern.

The Jerboa puzzle[1]

We will use a simple method that I will call the jerboa puzzle. What is jerboa? This is a small mouse like animal living on the Arabian Desert, and in a few other places. Jerboa has big ears and a puffy tail. When it runs on soft desert sand its tail leaves a path on the ground. Jerboas dig also holes in the ground, can jump, run away, etc.

Let us prepare a piece of desert for our jerboa. For this purpose we need a page of paper with a square grid. This can be, for example, a page from a school mathematics notebook. We can also construct our own grid on a large sheet of paper or cardboard. Then we mark on the grid two types of points – red ones and the yellow points. In fact we do not need yellow points. The red points will be enough. This means that if we draw our ribbon patterns by hand we should draw the red points only. This will save our time. Then we make a fence around the area where our jerboa will live and we will observe the jerboa paths in his small playground. For the start we will choose a very small

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[1] The method presented here is a modification of the method, that I created for drawing Celtic interlace patterns. You can find it in my book 'Sketches on geometry and art – between East and West' (in Polish). Originally the method did not have a special name. This document is the first time, when I use jerboa to draw interlace patterns. The jerboa method is simple enough to use it with primary school children or even much earlier.

area 4 by 4 cells. Later we will use larger and more complex areas. The sequence of images on the next page shows step-by-step trace of the jerboa's walks in his small playground.

Drawing by hand can be a great meditation opportunity, but it can be very tedious. The same tasks can be done with geometry software. Computer screen will replace our paper and custom tools will replace tedious drawing of points and paths. Here I show my works with Geometer's Sketchpad.

Project 2. Our first ribbon pattern

This project can be done by designing all necessary tools in a geometry program and then using them to produce the complete pattern. We will need a square grid and two tools – one to draw intersection of jerboa path, as well as another tool to draw path near the border where jerboa bounces from the fence.
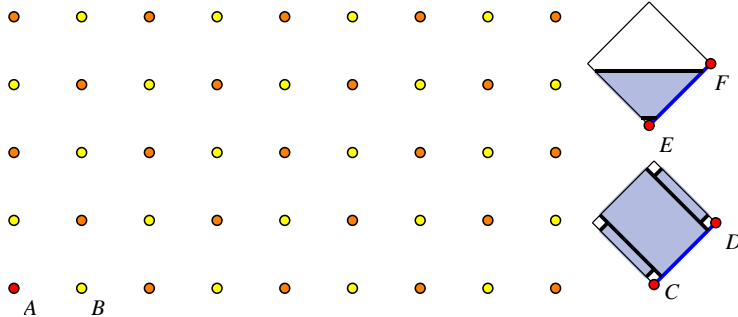
Tools necessary for this and following projects: a tool for making grid – its initial points are points 'A' and 'B'; a tool for creating crossing of the path – its initial points are points 'C' and 'D'; and a tool for border with initial points in 'E' and 'F'. The last two tools should be created separately, using the square tool from the previous project. The width of the band is equal 10/12 of the side of the square.
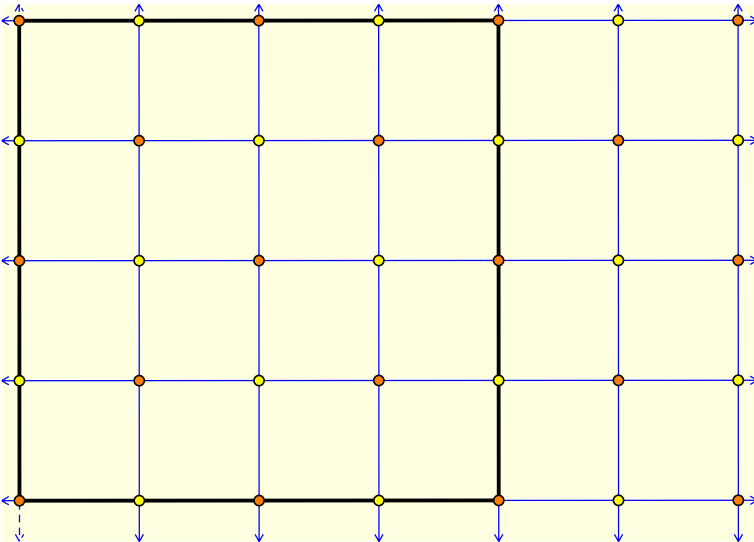

Here the jerboa's playground is a square area 4 by 4 cells. The corners of the fence are always located it red points. We can consider red points as stones on the desert or sticks regularly stuck in the sand.
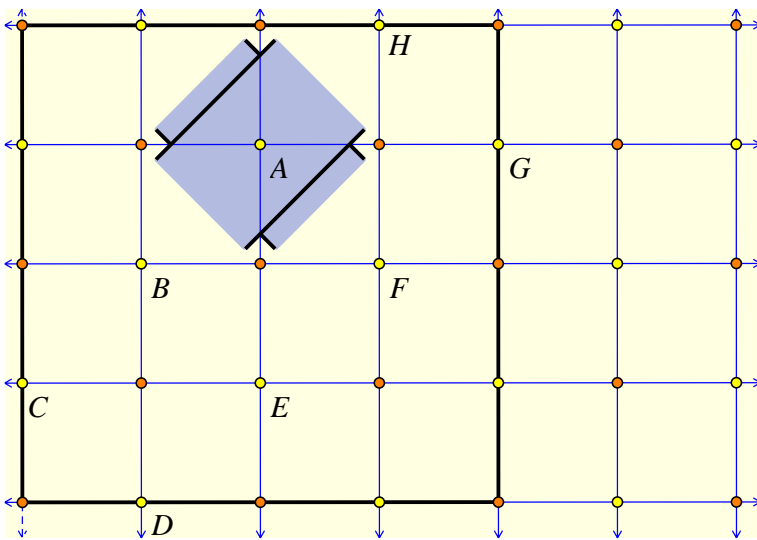
Suppose that the jerboa starts from a point A and wants to walk down-left. We will draw his path around point A. Simply take the square determined by the red stones around A and draw a fragment of the jerboa's path. The path can be wide or even wider than in our picture, or narrower, say ½ of the side of the square. Here we used custom tool to make this drawing.

The next jerboa step should be to the point B, but because in point A jerboa was on the surface, in point B he will go underground. Thus he should appear in the area near the point C.
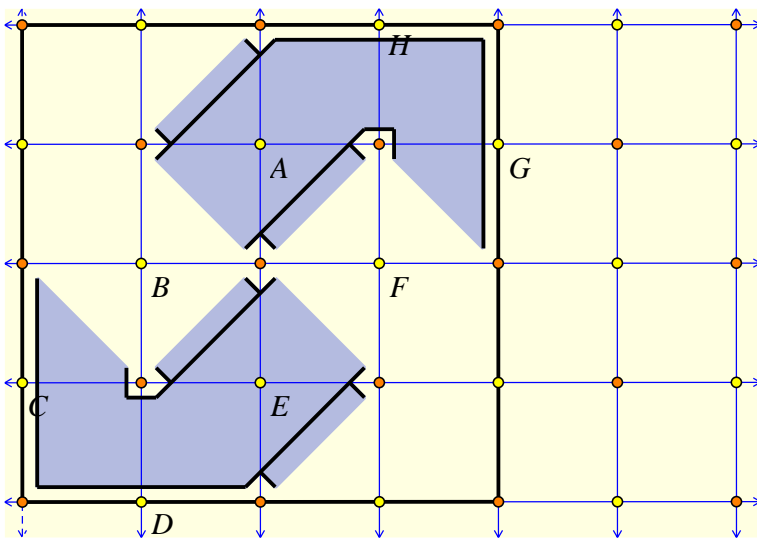
Here is shown the trace left by the jerboa when starting from the point A and going along path A, B, C, D, E, F, G, H, A. In points B and F the jerboa digs a tunnel underground. At all other points it is on the surface.

At points C, D, G, and H the jerboa bounces from the fence. These are not crossing points. Thus the jerboa is still on the surface.
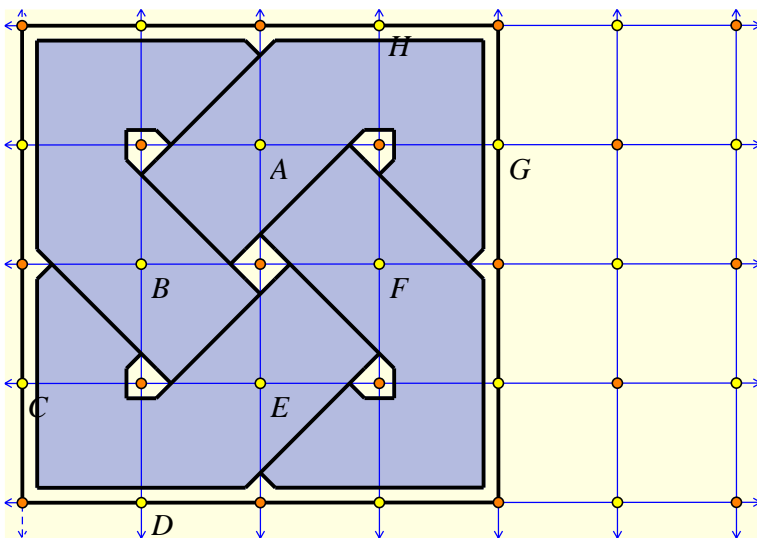
Here we show the second path left by another jerboa. This time the starting point is B and its path is perpendicular to the path of the previous jerboa.

NOTE: the jerboa path can be wider and this way we may obtain a style similar to figure 5. If the path is narrower then we may obtain a style similar to the one shown in figure 8. Such a style we will call the Alhambra style.
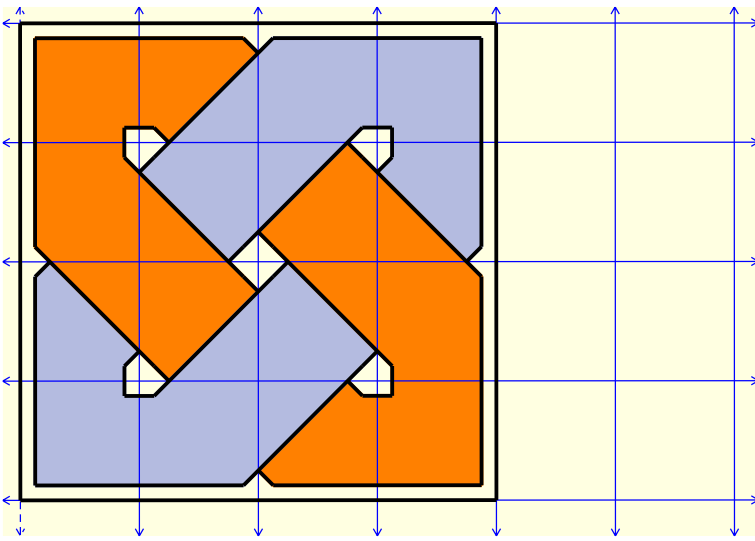
A color representation of the jerboa's path. We used two different colors to emphasize the structure of the pattern. Our pattern is designed from two closed paths.

Coloring ribbon patterns can be a nice adventure for our students. It is often the main question – how many different intersecting paths we can produce on a given bounded area? Another question that we could state here is – how the shape of the border influences the number of paths?

One can think about jerboa puzzle as a real puzzle made out of a number of square and triangular pieces. In fact the simplest jerboa puzzle will need two types of pieces like those that we used in this project. The short side of the triangle should be equal to the side of the square. In our project we used 8 triangles and 4 squares. For more complex examples we will need more pieces. Each time the number of triangles should be twice the number of squares or even larger. Later we will introduce a few new pieces for the jerboa puzzle. Let us summarize the rules for the jerboa puzzle:

1. The jerboa playground should be enclosed by a closed fence. If there are breaks in the fence then the jerboa can run away.

2. Each corner of the fence should be at a red point.

3. If it is possible, the jerboa should walk on a straight slant path towards the border, otherwise it can walk into any available space.

4. The jerboa turns only near the border (it bounces like a ball).

5. The jerboa goes alternately above or below the ground while passing through yellow points that are not on the border (points on the border are not crossing points).

Now, we are ready to create a few more ribbon designs.

## Project 3. Two different jerboa playgrounds

Draw a ribbon pattern using each of the two enclosed grids. Size of each rectangle is different and this way we can see how the shape of the rectangle influences the number of paths. Paint each path with different color. This will help you to count the number of paths.

When you will finish dealing with these two playgrounds design a few more rectangles and see how jerboa can walk on them. In this and all following projects you can ignore yellow points as well as the gridlines. The red points and boundaries is really all what you need.

2a   2b
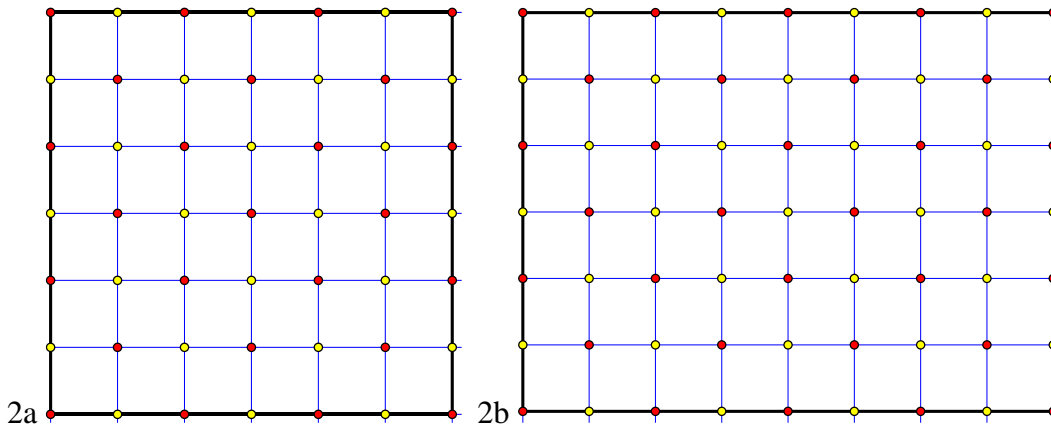
Project 4. Jerboa walks between complex borders

In our projects we used very simple shapes for the jerboa playground. However, the only restriction while marking the border is that each corner of the border should be at a red point. Thus we can make a very fancy border for jerboa walks. For example like the one in the enclosed picture.
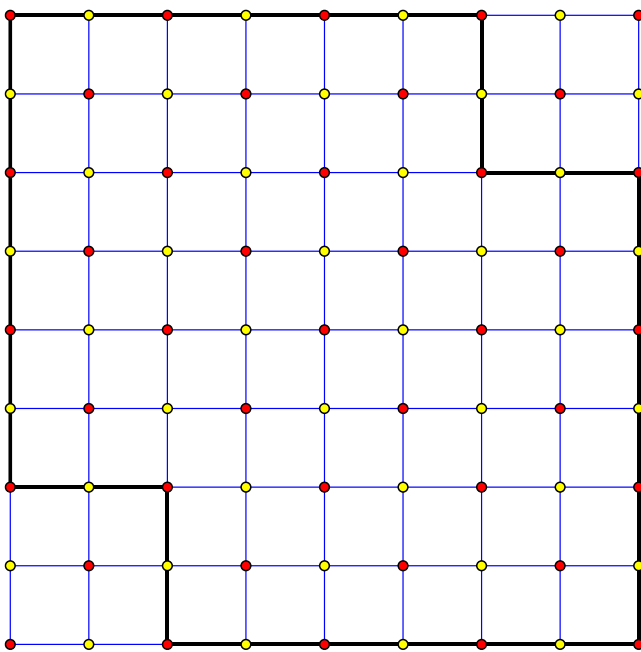

Try to develop a ribbon pattern using this playground. How many closed paths are there? Paint each of them using different color. Draw your own borders for a ribbon pattern.

The real adventure with jerboa walks starts when we will add some internal partitions for the jerboa walking area. The jerboa playground can be partitioned in many interesting ways. As we see it in the next project this will allow us to make even more interesting ribbon patterns.

Project 5. Ribbon patterns with partitions

Develop a ribbon pattern using the following area with partitions:
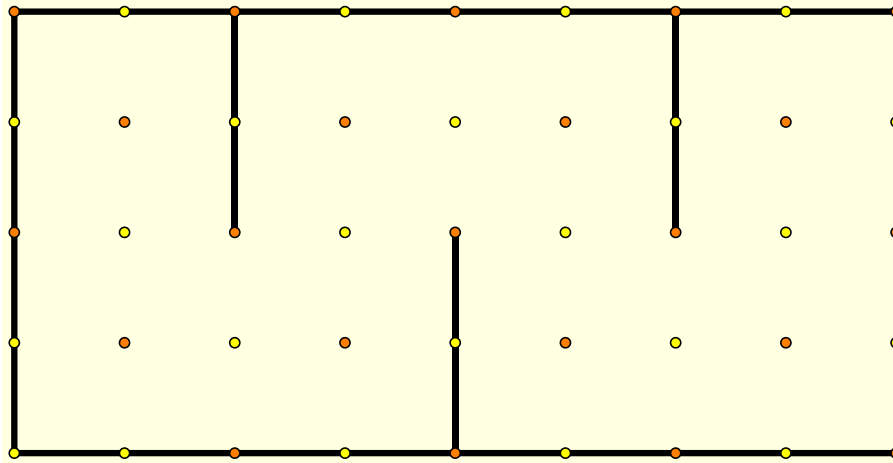
This area, 8 by 4 squares, can be expanded to the left and right as far as we want. The internal partitions placed at regular intervals force the jerboa to change direction every time he bounces from a partition or a boundary.
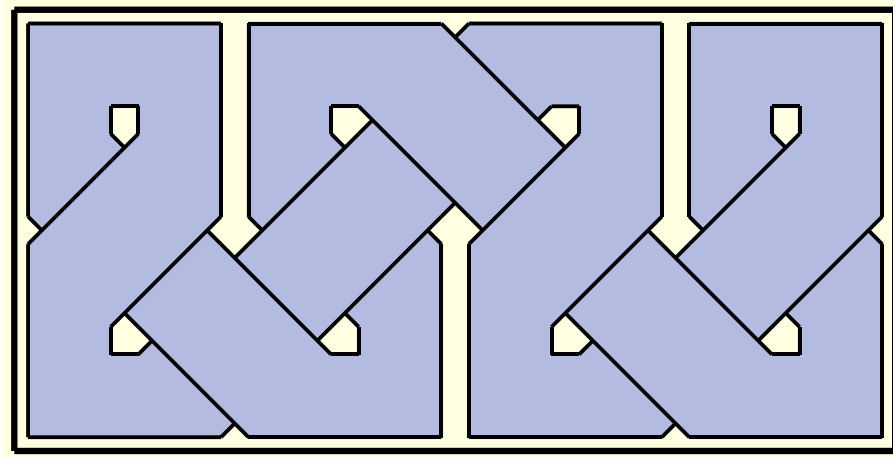

The ribbon pattern created using the above area. This pattern is one of the popular ribbon patterns in Islamic art. Usually it is rendered with wide paths almost without a space between them. This pattern is shown in the Inci A. Birol book (see [1]).

Projects 6.1 and 6.2. More complex partitions

Develop ribbon patterns using the areas shown in the enclosed pictures. Each one has its own interesting features. In the first one the large cross partition in the middle splits the whole pattern into multiple paths. Some of them are stack in one quarter of the area. Some other can go through each quarter. Why?

In the second area we force jerboa to go along lengthy horizontal corridors. This way we obtain interesting interlace effects and paths going through multiple sections of the area. How many different paths we will get in such case? What will happen if we add two more columns in the middle of this area, and extend horizontal partitions respectively?
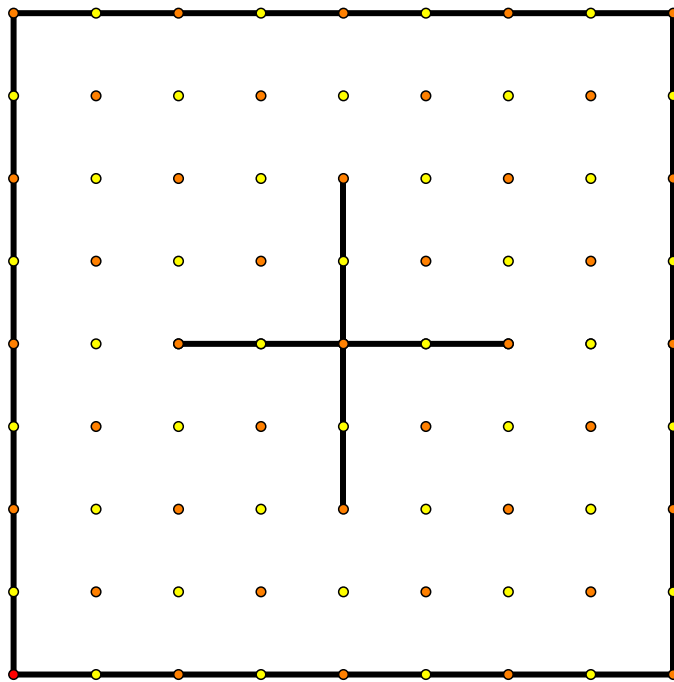
Project 6.1. Here the area for jerboa walks is 8 by 8 squares with large cross partition in the middle. This partition forces the jerboa to create a complex and very interesting pattern. We will use it later to introduce additional element to our jerboa puzzle (see Project 8).
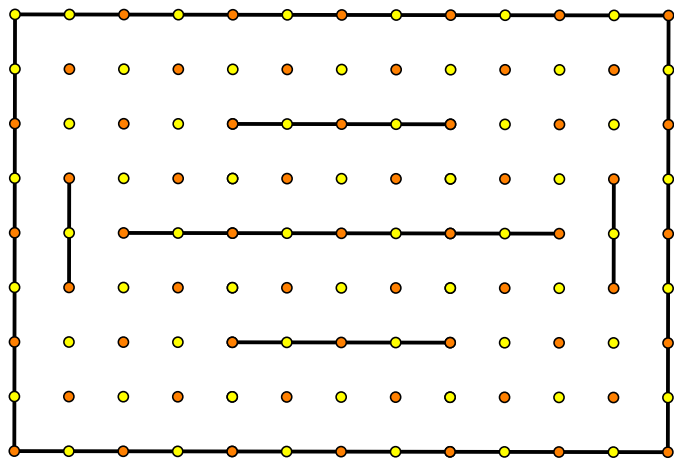
Project 6.2. This area can be extended to the right and left by using reflections about the vertical partitions on the right and left. This way we will obtain a ribbon pattern similar to the bottom part of the pattern shown on page 116 in Wade's book.

Project 7. Do it yourself

This time our task is different. We have a ribbon pattern shown in the picture. Our goal will be to develop grid, boundaries and partitions for this pattern. This pattern is similar to the one in project 5. Therefore, its grids and partitions should also be similar.
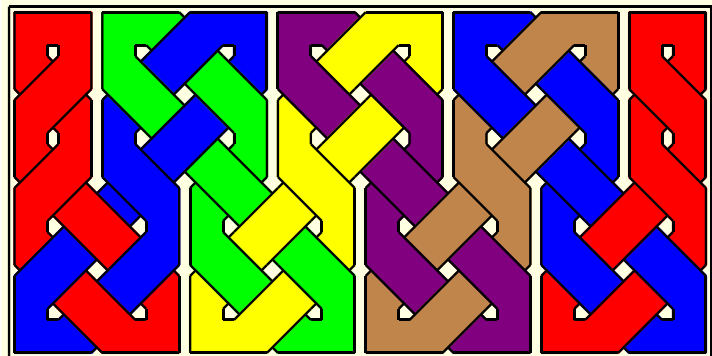
Project 8. Infinite ribbon pattern

Here is a more complex setup of partitions. Ribbon pattern based on this setup is often seen in countless places. The two dashed lines mean that the pattern can be extended to the right and left. The appearance of the pattern beyond these boundaries can be determined by a similar, or a completely different set of partitions. For example a pattern like this can go around a page with some text in the middle. Similar application we have seen already (fig. 6).
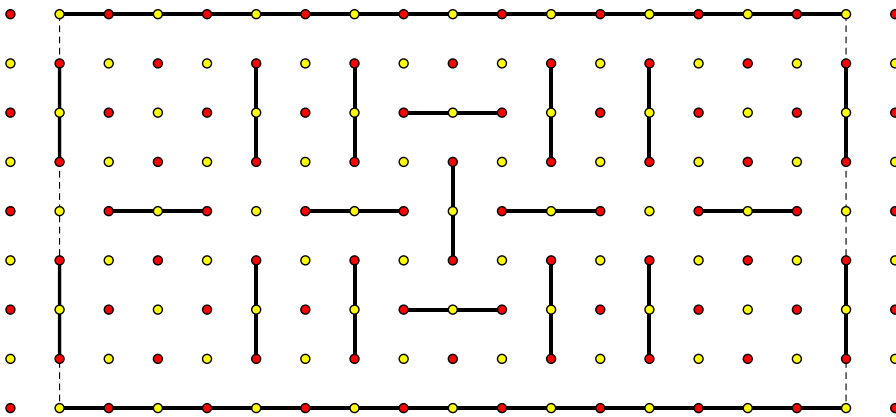

Dealing with irregular partitions

In ribbon patterns from Alhambra it happens from time to time that some red points are removed and some partitions are shortened in such a way that they connect a red point with a yellow point. In such case in the setup of our partitions we may have irregularities that will force the jerboa to make some unusual steps. One such situation is shown in the next example.

Project 9. Missing red point
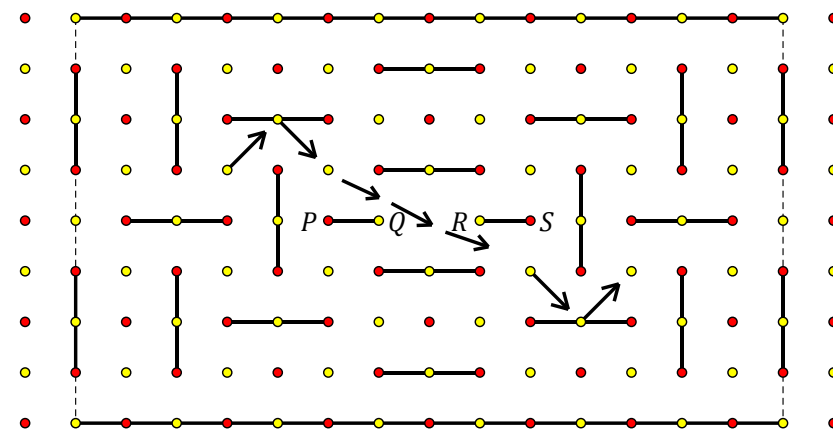
In this setting there is the missing red point between points Q and R, and partitions PQ and RS are shortened. This means that the jerboa passing near point Q will no longer be forced to go horizontally and after passing point Q he will slip down. Then he will face another horizontal partition that will force him to go horizontally for a short while. The jerboa path may look like the one shown on the following picture
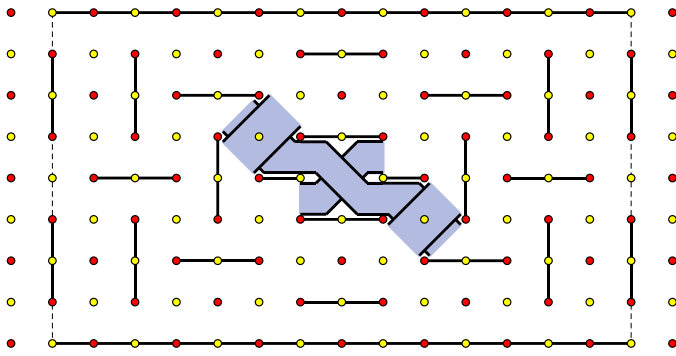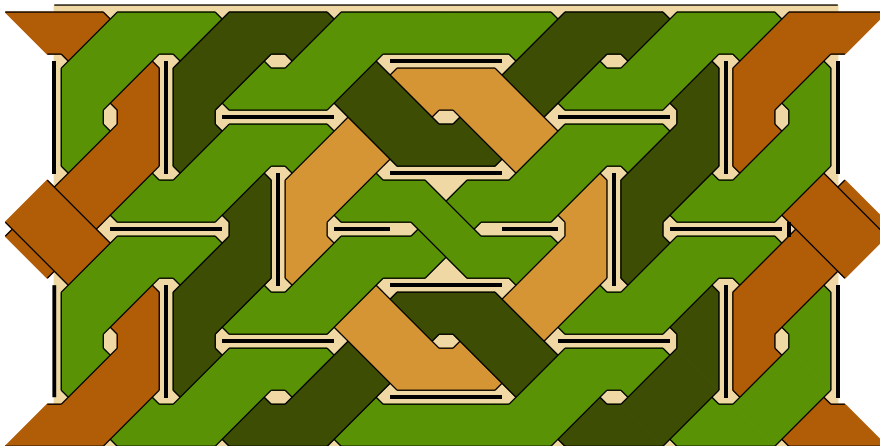
In order to fill the area between points Q and R we needed three new elements: A – a triangle, B – a mirror of the triangle A, and X the crossing.
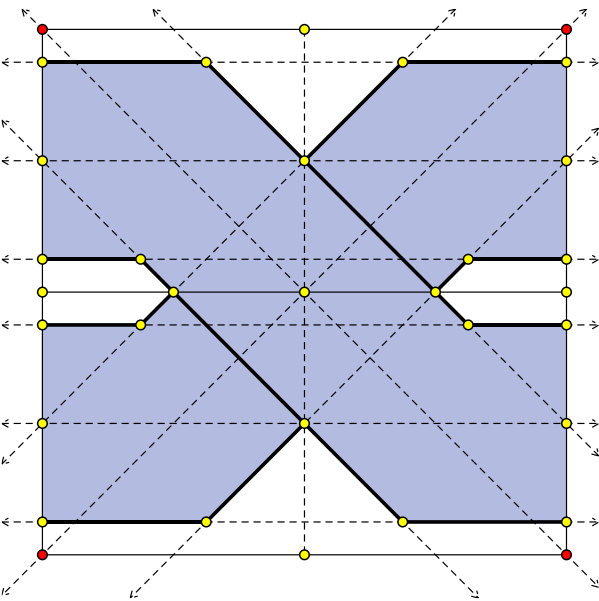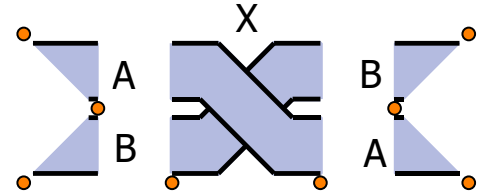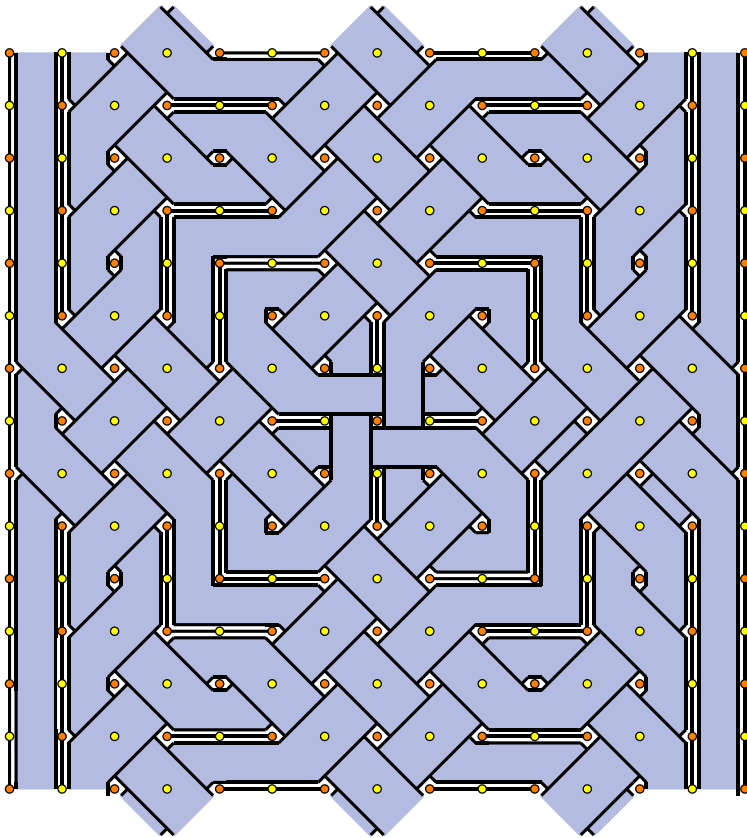

Construction of the element X – the crossing of two paths – right-down and left-down.

Note, one can simplify this situation by creating only one tool that is combined version of the three tools – A, B and X. This can be easily done by developing a direct construction of such new tool, or make, a tool using these three tools together.

There are no more surprises in this setting. The remaining path or paths can be developed in a similar way as in all previous projects. Try to finish this pattern and make it colorful like many other patterns from the Alhambra. Note – this pattern is similar to the fragment of the Alhambra pattern shown in the figure 8. Below I show the final result that could be produced in GSP with our tools. Colors were added later.

Project 10. Gap around a red point.

Let us consider another setting of partitions. In this case partitions near the center of the grid were shortened but the central point was not removed (see Project 9). This forces the jerboa to continue his walk on horizontally or vertically, otherwise he will have to cross through the red point. In such case we will obtain a new type of crossing in the middle of the pattern. Its paths will be horizontal and vertical – not slant like we had it before. Try to finish this pattern. Just remember the rule 5 – the jerboa's path goes alternately above or below the ground.

Two versions of the finished ribbon pattern from project 8.

One with points and partitions (left).

The other one, below, with colors similar to the colors used in the book 'L'Ornement Polychrome' by August Racinet, Paris 1888.
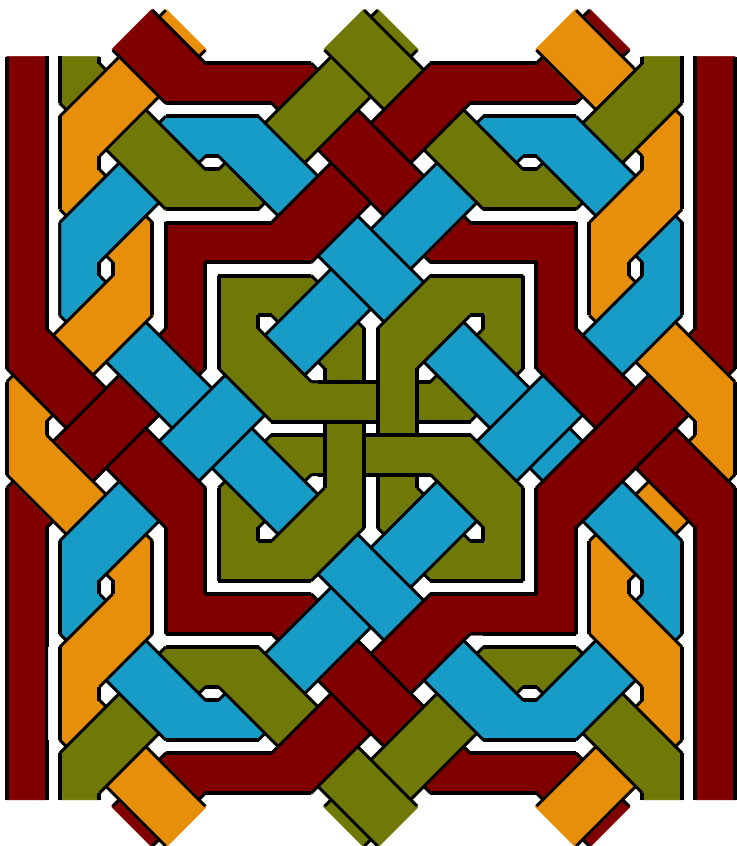
A matter of style

All projects in this paper have used reasonably wide paths for the jerboa walks. This was convenient for constructing in a consistent manner the square as well as the triangular elements, and drawing the ribbon patterns. However, in our designs there is one thing that is worth spotting. The slant paths are a bit wider than horizontal or vertical paths.

Fig. 11. *In the presented approach horizontal and vertical paths are slightly narrower than the slant paths.*

This can be illustrated by the drawing. Here the width of slant paths is equal to ½ of the length of the side of the square ABCD. At the same time the width of the horizontal path is equal to ½ of the distance between points A and E, which is equal to AB/√2=0.707AB.
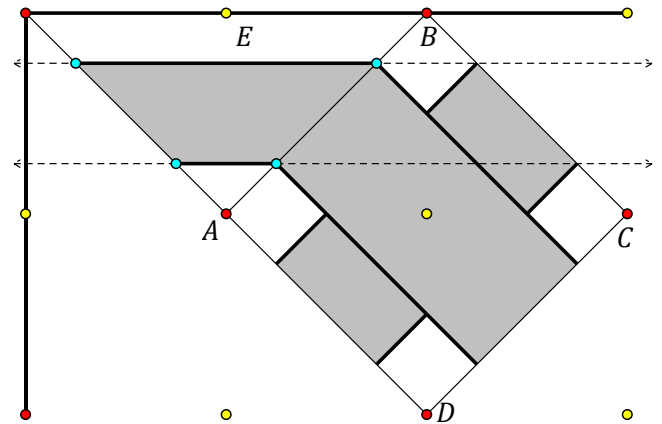

In a few examples from Islamic art we see that in some of them paths are as wide as possible – just touching the big dots of the grid. In the western Islamic art paths are narrower – just about ½ of the distance between two neighboring red points. In such designs paths, slant as well as horizontal and vertical, have the same width. This was not possible in our approach. Thus if we want to make designs identical with Alhambra designs we will have to modify our drawing methods. This can be done in a few ways. One of them we show below.

Fig. 12. *An improved method for drawing horizontal and vertical paths.*

In this method the width of slant paths is equal ½ of the length of the side of the square ABCD. The horizontal path has exactly the same width. But in order to achieve this effect we had to change the way how we draw both the triangular elements of the ribbon pattern and the square elements touching triangular elements.

Finally, let us notice that in this improved method in order to draw longer horizontal or vertical paths we will need three different triangular elements.
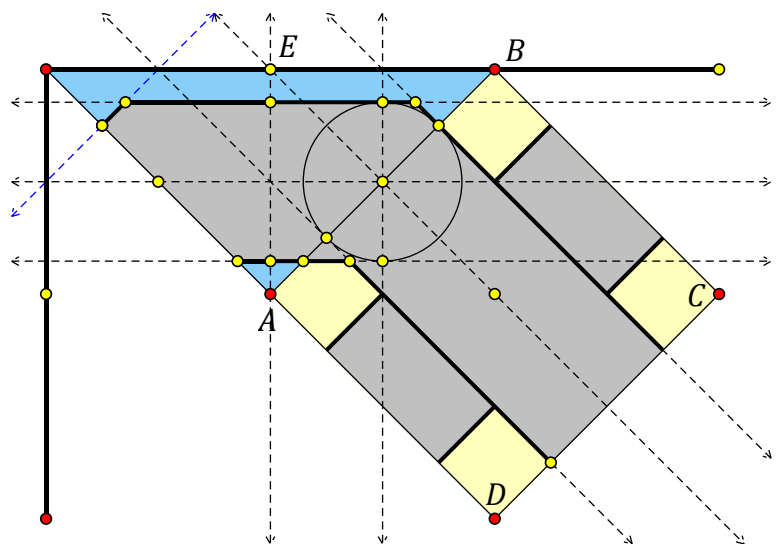
Now it is your turn. On your computer screen create a large grid, e.g. 21 by 21 squares, draw a border – it can be very fancy, and create a setup of partitions. Partitions can be very simple or quite complicated. This is up to you. Then develop a ribbon pattern using your setup. At the end paint it in your favorite way. It can be traditional coloring system similar to the colors used in the Alhambra or something very modern. Again, the choice is yours.
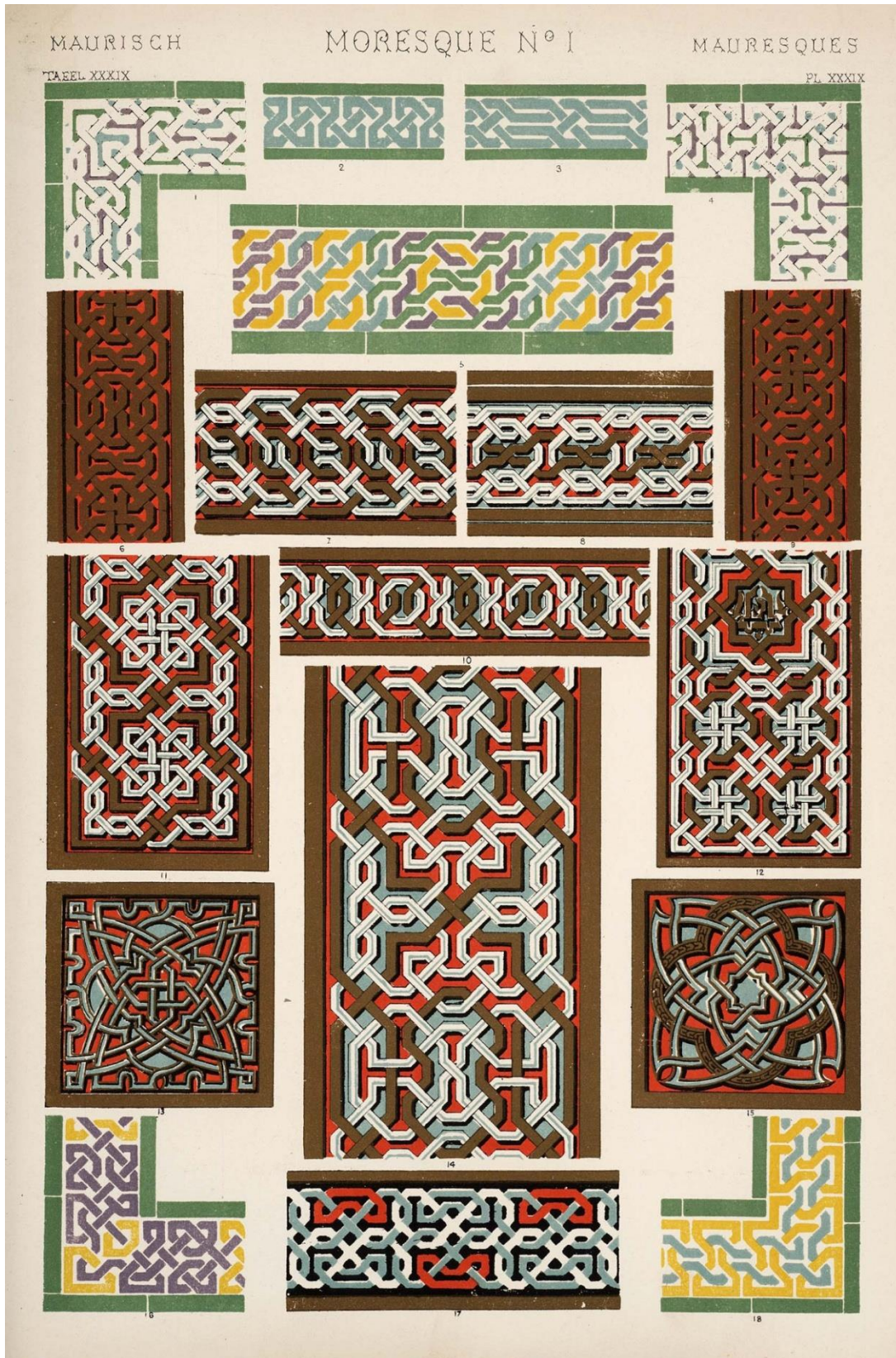
Fig. 13. *Page with ribbon patterns from the book The Grammar of the Ornament by Owen Jones (see [10]). Any of these patterns can be used for a classroom project with GSP or GeoGebra.*

Web Sketchpad Ribbon Patterns Toolbox

For the purpose of exploring and experimenting with ribbon patterns we can use a Ribbon Patterns Toolbox that was created with the use of Geometer's Sketchpad and Web Sketchpad The toolbox is available by following link (https://symmetrica.wordpress.com/gsp/) or from the EJMT web site.

Web Sketchpad is a utility developed by the GSP team at McGraw-Hill Education. WSP is still in beta phase.

Description of how to use Ribbon Pattern Toolbox is shown in the figure enclosed below. Coloring in this toolbox is different than in this paper. I wanted to match colors with the WSP color palette. Grids and yellow points are enclosed only in a few pages. I omitted them on pages with more advanced examples. The two curved arrows on left-top of WSP window can be used to undo and redo operations from the last one backwards. At the very moment WSP does not allow us to save our designs. Therefore, if you developed something really interesting, you have to capture the image on your screen using [PrtScr] key on your keyboard and paste the image into any graphics program.
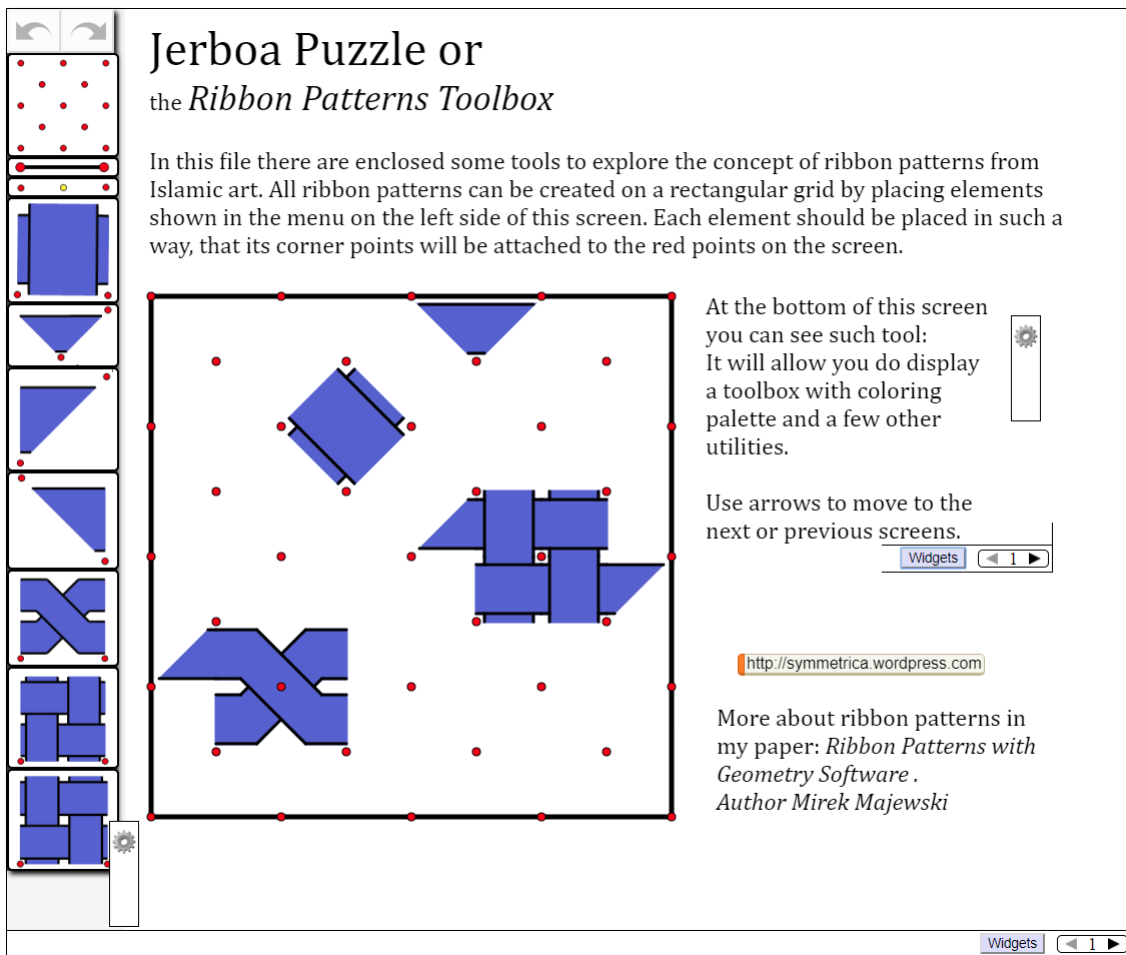

Fig. 14. *Window of the online Ribbon Pattern Toolbox*

References

[1]   Birol Inci A., *Türk Tezyînî Sanatlarında, DESEN TASARIMI, çizim tekniĝi ve çeşitleri* (Turkish ornamental art, Pattern Design, drawing techniques and types), Kubbealti, Istanbul, 2012.

[2]   Burckhard Titus, *Art of Islam, Language and Meaning*, Commemorative Edition, World Wisdom, Inc., 2009.

[3]   Calvert Albert F., *The Alhambra*, London, 1904.

[4]   Castera Jean-Marc, *Arabesques for Abu Dhabi*, Symmetry: Culture and Science Vol. 19, 2008.

[5]   Demiriz Yıldız, *Islam sanatinda, Geometrik Susleme, bir Envanter Denemesi* (Islamic art, geometric decoration, an inventory attempt), Iknici Basım, Istanbul, 2004.

[6]   Dye Daniel Sheets, *Chinese Lattice Designs*, Dover Publications, Inc., New York, 1974.

[7]   Jones Owen, *The Grammar of Ornament*, Bernard Quaritch, London, 1910.

[8]   Ремпелъ Л.И., *Искусство Среднего Востока* (The Art of the Middle East), Советский Художник, Москва, 1978.

[9]   Sutton Daud, *Islamic Design – a Genius for Geometry*, Walker & Company, New York, 2007.

[10] Wade David, *Pattern in Islamic Art*, The Overlook Press, Woodstock, New York, 1976.

[11] Wilson Eva, *Islamic Designs*, British Museum Pattern books, The British Museum Press, London, seventeenth edition, 2009.

[12] Graph paper web site: http://incompetech.com/graphpaper/, this is the place where you can download free graph paper for this paper.

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